



## Need of Fairness in Ranking Algorithms

Rankings are ubiquitous on online platforms and quintessential in finding relevant information quickly [Liu09; MRS10; ER15; Nob18; BW21]

Without fairness consideration, ranking algorithms output rankings that (1) **skew representations** of **social groups**—altering perception and polarizing opinions [ER15; KMM15] and (2) **skew exposure** at an **individual level**—leading to denial of economic opportunities to users [Han+17]

## Group Fairness Constraints Alone Are Insufficient

A growing number algorithms try to output “fair” rankings by ensuring a desired representation of each protected group [PSK21; ZYS22]

While **group fairness** can mitigate under-representation across groups, they **may not mitigate** (and **can even exacerbate**) **harms at individual level**

E.g., specific individuals (whose utilities have high uncertainty) may be systematically assigned to poorer positions [Han+17; SKJ21]

## Individual Fairness Requires Stochasticity

Recent work in fair ranking proposes to incorporate **fairness, from the individuals' perspective**, in rankings [SJ18; BGW18; SKJ21]. At a high level, they aim to equalize the exposure of individuals with similar “utilities”

**Problem:** Since exposure is linked to the position of individuals in the ranking, a single deterministic ranking cannot satisfy individual fairness

**Example:** For instance, suppose individuals  $i_1$  and  $i_2$  have similar utilities. Any deterministic ranking, must place one before the other, thereby imparting a systematically higher exposure to the earlier individual.

**Idea:** Introduce **stochasticity** in the output rankings [SJ18; BGW18; SKJ21]

However, satisfying individual fairness constraints alone, may introduce **misrepresentation at the group-level**

**Goal:** Output rankings satisfying both individual and group constraints

## Model of Fair Ranking

**Ranking** [MRS10; Liu09]:

- There are  $m$  individuals or items and  $n$  positions
- Placing the  $i$ -th item at the  $j$ -th position gives  $\rho_i \cdot v_j$  utility

**Output:** A ranking of  $n$  items with the highest utility

**Fair Ranking** [YS17; CSV18; SJ18...]:

**Protected groups:** A Laminar family of sets  $G_1, G_2, \dots, G_p \subseteq \{1, 2, \dots, m\}$

**Blocks:**  $B_1, B_2, \dots, B_q \subseteq \{1, 2, \dots, n\}$

Disjoint subsets of positions (e.g., different pages in web search)

**Group fairness constraints:** **Lower/upper bounds** specified by  $L/U$

Require at least  $L_{k\ell}$  and at most  $U_{k\ell}$  items from  $G_\ell$  in  $B_k$

E.g.,  $L_{k\ell} = U_{k\ell} = |B_k|/2 \Rightarrow$  Equal representation constraints

**Individual fairness constraints:** **Lower/upper bounds** specified by  $C/A$

Ensure item  $i$  appears in  $B_k$  with probability at least  $C_{ik}$  and at most  $A_{ik}$

**Output:** Ranking maximizing expected utility subject to fairness constraints

## Existing Algorithms Can Violate Group Fairness

Frameworks of some recent works can be adapted to incorporate both individual and group fairness constraints [SJ19; SJ18; SKJ21]

They solve a linear program (LP), whose solution is a **distribution over rankings** that (1) **satisfies individual fairness constraints** and (2) **rankings sampled from it** satisfy the **group fairness constraints in expectation**

**Problem:** Even though sampled rankings satisfy group fairness constraints in aggregate, **each output ranking** can violate **group fairness constraints**

## Theoretical Guarantees on Sampling Group-Fair Rankings from Individually-Fair Distributions

An efficient algorithm that samples rankings from an individually-fair distribution while ensuring that every output ranking is group fair

**Main Result.** There is a polynomial time algorithm, that given  $L, U, A, C$  specifying fairness constraints, and  $\rho$  and  $v$  specifying utilities, outputs a ranking  $R$  sampled from distribution  $\mathcal{D}$  such that

1.  $\mathcal{D}$  satisfies  $(C, A)$ -individual-fairness constraints; and
2.  $R$  satisfies  $(L, U)$ -group-fairness constraints.

The expected utility of  $R$  is at least  $\alpha$ -times the optimal, where, e.g.,

$$\alpha \geq \frac{v_1 + v_2 + \dots + v_k}{k \cdot v_1}.$$

We present tighter bounds on  $\alpha$  in the paper.  $\alpha$  approaches 1 as either (1) the range of utilities or (2) the range of position-discounts shrinks.

E.g., with DCG position discounts [JK02] and  $k = 1, 2, 3$ ,  $\alpha$  is  $\geq 0.8, 0.7, 0.6 \dots$

## Technical Approach and Challenges

Existing works [SJ19; SJ18; SKJ21] use the **Birkhoff-von-Neumann algorithm** [Bru82] to sample rankings from **individually-fair distribution** such that they satisfy **group fairness constraints in aggregate**

**Idea:** If the polytope of group-fair rankings is **integral**, one can use adaptations of the Birkhoff-von-Neumann algorithm

**Problem:** Group-fair rankings form a **non-integral polytope** (Section A)

**Idea:** Consider a family of “**coarse rankings**.” A coarse ranking is the same as a ranking except that it does **not order items within blocks**

**Fact:** The **polytope of coarse rankings is integral** [PLN22] and, hence, one can use adaptations of the Birkhoff-von-Neumann algorithm

**Algorithm:** (1) Sample a group-fair coarse ranking from an individually-fair distribution and (2) Convert the sampled coarse ranking to a proper ranking (this step loses at most  $(1 - \alpha)$ -fraction of the utility)

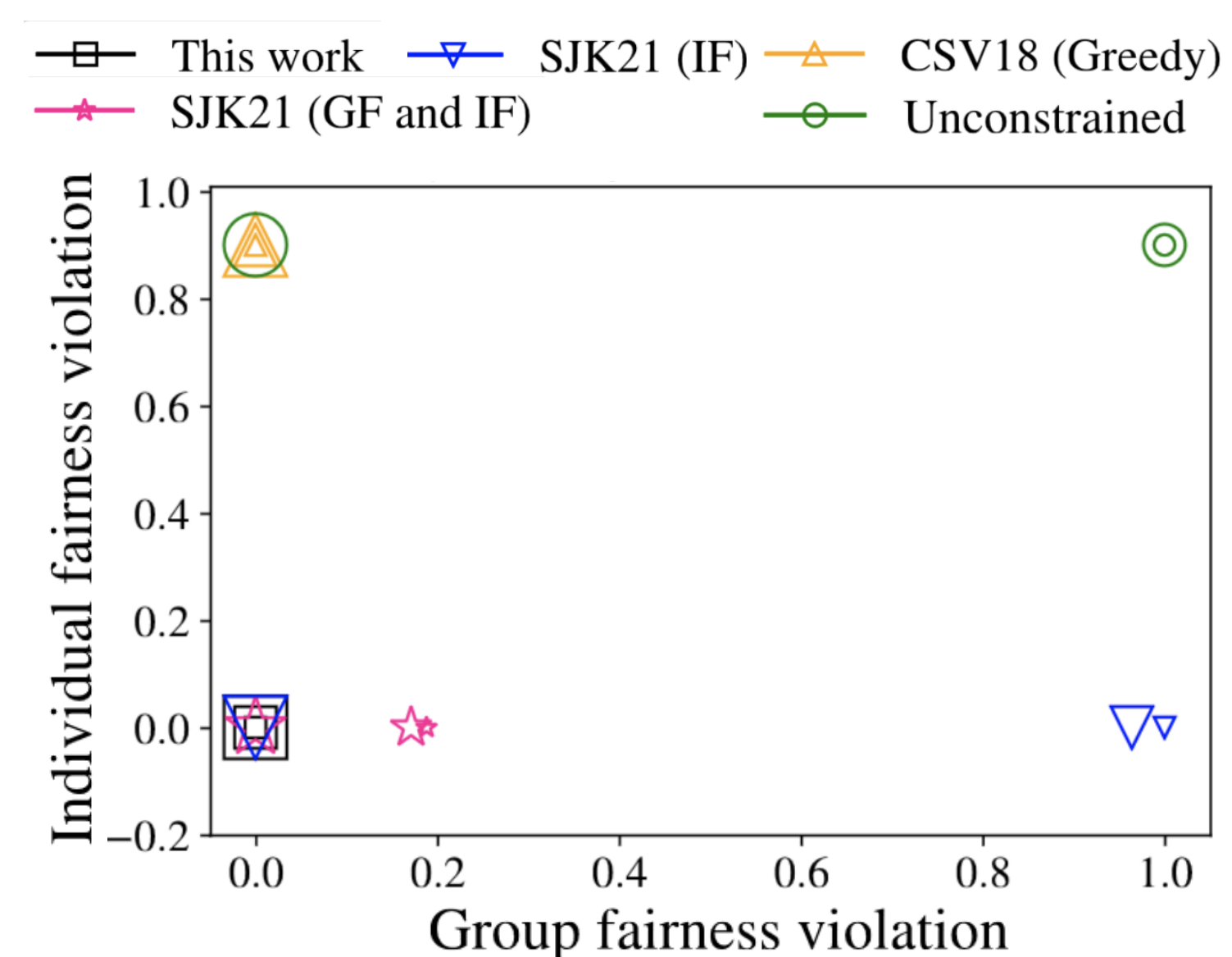
## Empirical Results on Real-World Data

**Data:** Occupations data [CK20] which contains **top 100 Google image search results** for **96 occupation** related queries. The data has crowd-sourced gender attributes—which we use as ground truth

**Notion of Fairness:** Equal representation across genders and individuals receive at least a certain exposure determined by their utility

**Metrics:** (1) Probability of violating equal representation, (2) deviation from the specified individual fairness constraints, and (3) DCG-based utility

**Main Observation:** Our algorithm outputs rankings which **always satisfy the specified fairness constraints**; while **losing  $\leq 6\%$  utility** with respect to baselines



The paper also has empirical results with other datasets

## Limitations and Future Work

We consider the utility of a ranking to be a linear function of the utilities items in the ranking. While this captures many applications [ZYS22a; ZYS22b; PSK21], in some applications, the utilities may be non-linear [Agr+09; Asa+22; KRT22].

Our algorithm works for Laminar family of groups, but not for an arbitrary family of protected groups